Mathematica 11.3 Integration Test Results

Test results for the 58 problems in "7.1.4b (f x)^m (d+e x^2)^p (a+b arcsinh(c x))^n.m"

Problem 6: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b\,\text{ArcSinh}[c\,x]}{d+e\,x^2}\,dx$$

Optimal (type 4, 485 leaves, 18 steps):

$$\frac{\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcSinh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d+e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcSinh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d+e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}+$$
$$\frac{\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcSinh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d+e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\,\text{ArcSinh}[c\,x]\right)\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcSinh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d+e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}-$$
$$\frac{b\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcSinh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d+e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}+\frac{b\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcSinh}[c\,x]}}{c\,\sqrt{-d}-\sqrt{-c^2\,d+e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}-$$
$$\frac{b\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcSinh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d+e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}+\frac{b\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcSinh}[c\,x]}}{c\,\sqrt{-d}+\sqrt{-c^2\,d+e}}\right]}{2\,\sqrt{-d}\,\sqrt{e}}$$

Result (type 4, 775 leaves):

$$\frac{1}{16\,\sqrt{d}\,\sqrt{e}}\left(16\,a\,\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]+\right.$$
$$4\,b\left(8\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{ArcTan}\left[\frac{\left(c\,\sqrt{d}-\sqrt{e}\right)\,\text{Cot}\left[\frac{1}{4}\,\left(\pi+2\,i\,\text{ArcSinh}[c\,x]\right)\right]}{\sqrt{c^2\,d-e}}\right]-\right.$$
$$8\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{c\,\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{ArcTan}\left[\frac{\left(c\,\sqrt{d}+\sqrt{e}\right)\,\text{Cot}\left[\frac{1}{4}\,\left(\pi+2\,i\,\text{ArcSinh}[c\,x]\right)\right]}{\sqrt{c^2\,d-e}}\right]+$$

$$\left(\pi+4\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]-2\,i\,\text{ArcSinh}[c\,x]\right)\text{Log}\left[1-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d-e}\right)e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\right]-$$

$$\left(\pi+4\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]-2\,i\,\text{ArcSinh}[c\,x]\right)\text{Log}\left[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d-e}\right)e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\right]-$$

$$\left(\pi-4\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]-2\,i\,\text{ArcSinh}[c\,x]\right)\text{Log}\left[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d-e}\right)e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\right]+$$

$$\left(\pi-4\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]-2\,i\,\text{ArcSinh}[c\,x]\right)\text{Log}\left[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2 d-e}\right)e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\right]+$$

$$\left(\pi-2\,i\,\text{ArcSinh}[c\,x]\right)\text{Log}\left[c\left(\sqrt{d}-i\sqrt{e}\,x\right)\right]+2\,i\,\text{ArcSinh}[c\,x]\,\text{Log}\left[c\left(\sqrt{d}-i\sqrt{e}\,x\right)\right]-$$

$$\left(\pi-2\,i\,\text{ArcSinh}[c\,x]\right)\text{Log}\left[c\left(\sqrt{d}+i\sqrt{e}\,x\right)\right]-2\,i\,\text{ArcSinh}[c\,x]\,\text{Log}\left[c\left(\sqrt{d}+i\sqrt{e}\,x\right)\right]-$$

$$2\,i\left(\text{PolyLog}\left[2,\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d-e}\right)e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\right]+\text{PolyLog}\left[2,-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d-e}\right)e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\right]\right)+$$

$$2\,i\left(\text{PolyLog}\left[2,-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d-e}\right)e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\right]+\text{PolyLog}\left[2,\frac{i\left(c\sqrt{d}+\sqrt{c^2 d-e}\right)e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\right]\right)\Bigg)\Bigg)$$

Problem 7: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,\text{ArcSinh}[c\,x]}{\left(d+e\,x^2\right)^2}\,dx$$

Optimal (type 4, 707 leaves, 26 steps):

$$-\frac{a+b\,\text{ArcSinh}[c\,x]}{4\,d\,\sqrt{e}\,\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{a+b\,\text{ArcSinh}[c\,x]}{4\,d\,\sqrt{e}\,\left(\sqrt{-d}+\sqrt{e}\,x\right)}-\frac{b\,c\,\text{ArcTan}\left[\frac{\sqrt{e}-c^2\sqrt{-d}\,x}{\sqrt{c^2 d-e}\,\sqrt{1+c^2x^2}}\right]}{4\,d\,\sqrt{c^2 d-e}\,\sqrt{e}}-$$

$$\frac{b\,c\,\text{ArcTan}\left[\frac{\sqrt{e}+c^2\sqrt{-d}\,x}{\sqrt{c^2 d-e}\,\sqrt{1+c^2x^2}}\right]}{4\,d\,\sqrt{c^2 d-e}\,\sqrt{e}}-\frac{\left(a+b\,\text{ArcSinh}[c\,x]\right)\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcSinh}[c\,x]}}{c\sqrt{-d}-\sqrt{-c^2 d+e}}\right]}{4\,(-d)^{3/2}\sqrt{e}}+$$

$$\frac{\left(a+b\,\text{ArcSinh}[c\,x]\right)\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcSinh}[c\,x]}}{c\sqrt{-d}-\sqrt{-c^2 d+e}}\right]}{4\,(-d)^{3/2}\sqrt{e}}-\frac{\left(a+b\,\text{ArcSinh}[c\,x]\right)\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcSinh}[c\,x]}}{c\sqrt{-d}+\sqrt{-c^2 d+e}}\right]}{4\,(-d)^{3/2}\sqrt{e}}+$$

$$\frac{\left(a+b\,\text{ArcSinh}[c\,x]\right)\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcSinh}[c\,x]}}{c\sqrt{-d}+\sqrt{-c^2 d+e}}\right]}{4\,(-d)^{3/2}\sqrt{e}}+\frac{b\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcSinh}[c\,x]}}{c\sqrt{-d}-\sqrt{-c^2 d+e}}\right]}{4\,(-d)^{3/2}\sqrt{e}}-$$

$$\frac{b\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcSinh}[c\,x]}}{c\sqrt{-d}-\sqrt{-c^2 d+e}}\right]}{4\,(-d)^{3/2}\sqrt{e}}+\frac{b\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcSinh}[c\,x]}}{c\sqrt{-d}+\sqrt{-c^2 d+e}}\right]}{4\,(-d)^{3/2}\sqrt{e}}-\frac{b\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcSinh}[c\,x]}}{c\sqrt{-d}+\sqrt{-c^2 d+e}}\right]}{4\,(-d)^{3/2}\sqrt{e}}$$

Result (type 4, 1129 leaves):

$$\frac{a\,x}{2\,d\,\left(d+e\,x^2\right)}+\frac{a\,\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]}{2\,d^{3/2}\sqrt{e}}+$$

$$b\left(-\frac{-\frac{\text{ArcSinh}[c\,x]}{i\sqrt{d}+\sqrt{e}\,x}-\frac{c\,\text{Log}\left[\frac{2\,e\left(\sqrt{e}-i\,c^2\sqrt{d}\,x+\sqrt{-c^2 d+e}\,\sqrt{1+c^2x^2}\right)}{c\sqrt{-c^2 d+e}\left(i\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d+e}}}{4\,d\,\sqrt{e}}+\frac{\frac{\text{ArcSinh}[c\,x]}{-i\sqrt{d}+\sqrt{e}\,x}+\frac{c\,\text{Log}\left[-\frac{2\,e\left(\sqrt{e}+i\,c^2\sqrt{d}\,x+\sqrt{-c^2 d+e}\,\sqrt{1+c^2x^2}\right)}{c\sqrt{-c^2 d+e}\left(-i\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d+e}}}{4\,d\,\sqrt{e}}+\right.$$

$$\frac{1}{32\,d^{3/2}\sqrt{e}}\left(-i\left(\pi-2\,i\,\text{ArcSinh}[c\,x]\right)^2+32\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\right.$$

$$\text{ArcTan}\left[\frac{\left(c\sqrt{d}-\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcSinh}[c\,x]\right)\right]}{\sqrt{c^2 d-e}}\right]+4\left(\pi+4\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]-\right.$$

$$2\,i\,\text{ArcSinh}[c\,x]\Big)\,\text{Log}\Big[1-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d-e}\right)e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\Big]+4$$

$$\left(\pi-4\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]-2\,i\,\text{ArcSinh}[c\,x]\right)\text{Log}\Big[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2 d-e}\right)e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\Big]-$$

$$4\left(\pi-2\,i\,\text{ArcSinh}[c\,x]\right)\text{Log}\Big[c\sqrt{d}+i\,c\sqrt{e}\,x\Big]-8\,i\,\text{ArcSinh}[c\,x]\,\text{Log}\Big[c\sqrt{d}+i\,c\sqrt{e}\,x\Big]-$$

$$8\,i\left(\text{PolyLog}\Big[2,\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d-e}\right)e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\Big]+\right.$$

$$\left.\left.\text{PolyLog}\Big[2,-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d-e}\right)e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\Big]\right)\right)+$$

$$\frac{1}{32\,d^{3/2}\sqrt{e}}\left(i\left(\pi-2\,i\,\text{ArcSinh}[c\,x]\right)^2-32\,i\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]\right.$$

$$\text{ArcTan}\Big[\frac{\left(c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcSinh}[c\,x]\right)\right]}{\sqrt{c^2 d-e}}\Big]-$$

$$4\left(\pi+4\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]-2\,i\,\text{ArcSinh}[c\,x]\right)$$

$$\text{Log}\Big[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d-e}\right)e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\Big]-4$$

$$\left(\pi-4\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\Big]-2\,i\,\text{ArcSinh}[c\,x]\right)\text{Log}\Big[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d-e}\right)e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\Big]+$$

$$4\left(\pi-2\,i\,\text{ArcSinh}[c\,x]\right)\text{Log}\Big[c\sqrt{d}-i\,c\sqrt{e}\,x\Big]+8\,i\,\text{ArcSinh}[c\,x]\,\text{Log}\Big[c\sqrt{d}-i\,c\sqrt{e}\,x\Big]+$$

$$8\,i\left(\text{PolyLog}\Big[2,-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d-e}\right)e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\Big]+\right.$$

$$\left.\left.\left.\text{PolyLog}\left[2,\frac{i\left(c\sqrt{d}+\sqrt{c^2 d-e}\right)e^{\text{ArcSinh}[c x]}}{\sqrt{e}}\right]\right)\right)\right)$$

Problem 12: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{(a+b\,\text{ArcSinh}[c x])^2}{d+e x^2}\,dx$$

Optimal (type 4, 739 leaves, 22 steps):

$$\frac{(a+b\,\text{ArcSinh}[c x])^2\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcSinh}[c x]}}{c\sqrt{-d}-\sqrt{-c^2 d+e}}\right]}{2\sqrt{-d}\sqrt{e}}-\frac{(a+b\,\text{ArcSinh}[c x])^2\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcSinh}[c x]}}{c\sqrt{-d}-\sqrt{-c^2 d+e}}\right]}{2\sqrt{-d}\sqrt{e}}+$$

$$\frac{(a+b\,\text{ArcSinh}[c x])^2\,\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcSinh}[c x]}}{c\sqrt{-d}+\sqrt{-c^2 d+e}}\right]}{2\sqrt{-d}\sqrt{e}}-\frac{(a+b\,\text{ArcSinh}[c x])^2\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcSinh}[c x]}}{c\sqrt{-d}+\sqrt{-c^2 d+e}}\right]}{2\sqrt{-d}\sqrt{e}}-$$

$$\frac{b\,(a+b\,\text{ArcSinh}[c x])\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcSinh}[c x]}}{c\sqrt{-d}-\sqrt{-c^2 d+e}}\right]}{\sqrt{-d}\sqrt{e}}+$$

$$\frac{b\,(a+b\,\text{ArcSinh}[c x])\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcSinh}[c x]}}{c\sqrt{-d}-\sqrt{-c^2 d+e}}\right]}{\sqrt{-d}\sqrt{e}}-$$

$$\frac{b\,(a+b\,\text{ArcSinh}[c x])\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcSinh}[c x]}}{c\sqrt{-d}+\sqrt{-c^2 d+e}}\right]}{\sqrt{-d}\sqrt{e}}+$$

$$\frac{b\,(a+b\,\text{ArcSinh}[c x])\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcSinh}[c x]}}{c\sqrt{-d}+\sqrt{-c^2 d+e}}\right]}{\sqrt{-d}\sqrt{e}}+\frac{b^2\,\text{PolyLog}\left[3,-\frac{\sqrt{e}\,e^{\text{ArcSinh}[c x]}}{c\sqrt{-d}-\sqrt{-c^2 d+e}}\right]}{\sqrt{-d}\sqrt{e}}-$$

$$\frac{b^2\,\text{PolyLog}\left[3,\frac{\sqrt{e}\,e^{\text{ArcSinh}[c x]}}{c\sqrt{-d}-\sqrt{-c^2 d+e}}\right]}{\sqrt{-d}\sqrt{e}}+\frac{b^2\,\text{PolyLog}\left[3,-\frac{\sqrt{e}\,e^{\text{ArcSinh}[c x]}}{c\sqrt{-d}+\sqrt{-c^2 d+e}}\right]}{\sqrt{-d}\sqrt{e}}-\frac{b^2\,\text{PolyLog}\left[3,\frac{\sqrt{e}\,e^{\text{ArcSinh}[c x]}}{c\sqrt{-d}+\sqrt{-c^2 d+e}}\right]}{\sqrt{-d}\sqrt{e}}$$

Result (type 4, 3196 leaves):

$$\frac{1}{8\sqrt{d}\sqrt{e}}\left(8 a^2\,\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]+\right.$$

$$4\,a\,b\left(8\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}-\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcSinh}[c\,x]\right)\right]}{\sqrt{c^2 d-e}}\right]-\right.$$

$$8\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcSinh}[c\,x]\right)\right]}{\sqrt{c^2 d-e}}\right]+$$

$$\left(\pi+4\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]-2\,i\,\text{ArcSinh}[c\,x]\right)\text{Log}\left[1-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d-e}\right)e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\right]-$$

$$\left(\pi+4\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]-2\,i\,\text{ArcSinh}[c\,x]\right)\text{Log}\left[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d-e}\right)e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\right]-$$

$$\left(\pi-4\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]-2\,i\,\text{ArcSinh}[c\,x]\right)\text{Log}\left[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d-e}\right)e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\right]+$$

$$\left(\pi-4\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]-2\,i\,\text{ArcSinh}[c\,x]\right)\text{Log}\left[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2 d-e}\right)e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\right]+$$

$$\left(\pi-2\,i\,\text{ArcSinh}[c\,x]\right)\text{Log}\left[c\left(\sqrt{d}-i\sqrt{e}\,x\right)\right]+2\,i\,\text{ArcSinh}[c\,x]\,\text{Log}\left[c\left(\sqrt{d}-i\sqrt{e}\,x\right)\right]-$$

$$\left(\pi-2\,i\,\text{ArcSinh}[c\,x]\right)\text{Log}\left[c\left(\sqrt{d}+i\sqrt{e}\,x\right)\right]-2\,i\,\text{ArcSinh}[c\,x]\,\text{Log}\left[c\left(\sqrt{d}+i\sqrt{e}\,x\right)\right]-$$

$$2\,i\left(\text{PolyLog}\left[2,\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d-e}\right)e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\right]+\right.$$

$$\left.\text{PolyLog}\left[2,-\frac{i\left(c\sqrt{d}+\sqrt{c^2 d-e}\right)e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\right]\right)+$$

$$2\,i\left(\text{PolyLog}\left[2,-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d-e}\right)e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\right]+\right.$$

$$\left.\left.\text{PolyLog}\left[2,\frac{i\left(c\sqrt{d}+\sqrt{c^2 d-e}\right)e^{\text{ArcSinh}[c x]}}{\sqrt{e}}\right]\right)\right)+4 b^2$$

$$\left(8 i \,\text{ArcSin}\left[\frac{\sqrt{1+\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{ArcSinh}[c x]\,\text{ArcTan}\left[\frac{\left(c\sqrt{d}-\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2 i\,\text{ArcSinh}[c x]\right)\right]}{\sqrt{c^2 d-e}}\right]-\right.$$

$$8 i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{ArcSinh}[c x]$$

$$\text{ArcTan}\left[\frac{\left(c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2 i\,\text{ArcSinh}[c x]\right)\right]}{\sqrt{c^2 d-e}}\right]-8 i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]$$

$$\text{ArcSinh}[c x]\,\text{ArcTan}\left[\left(\left(c\sqrt{d}-\sqrt{e}\right)\left(\text{Cosh}\left[\frac{1}{2}\text{ArcSinh}[c x]\right]-i\,\text{Sinh}\left[\frac{1}{2}\text{ArcSinh}[c x]\right]\right)\right)\Big/\right.$$

$$\left.\left(\sqrt{c^2 d-e}\left(\text{Cosh}\left[\frac{1}{2}\text{ArcSinh}[c x]\right]+i\,\text{Sinh}\left[\frac{1}{2}\text{ArcSinh}[c x]\right]\right)\right)\right]+$$

$$8 i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{ArcSinh}[c x]\,\text{ArcTan}\Big[$$

$$\left(\left(c\sqrt{d}+\sqrt{e}\right)\left(\text{Cosh}\left[\frac{1}{2}\text{ArcSinh}[c x]\right]-i\,\text{Sinh}\left[\frac{1}{2}\text{ArcSinh}[c x]\right]\right)\right)\Big/$$

$$\left.\left(\sqrt{c^2 d-e}\left(\text{Cosh}\left[\frac{1}{2}\text{ArcSinh}[c x]\right]+i\,\text{Sinh}\left[\frac{1}{2}\text{ArcSinh}[c x]\right]\right)\right)\right]+$$

$$\pi\,\text{ArcSinh}[c x]\,\text{Log}\left[1-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d-e}\right)e^{\text{ArcSinh}[c x]}}{\sqrt{e}}\right]+$$

$$4\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\text{ArcSinh}[c x]\,\text{Log}\left[1-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d-e}\right)e^{\text{ArcSinh}[c x]}}{\sqrt{e}}\right]-$$

$$i\,\text{ArcSinh}[c x]^2\,\text{Log}\left[1-\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d-e}\right)e^{\text{ArcSinh}[c x]}}{\sqrt{e}}\right]-$$

$$\pi\,\text{ArcSinh}[c x]\,\text{Log}\left[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2 d-e}\right)e^{\text{ArcSinh}[c x]}}{\sqrt{e}}\right]-$$

$$4\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{ArcSinh}[c\,x]\,\text{Log}\left[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2\,d-e}\right)\,e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\right]+$$

$$i\,\text{ArcSinh}[c\,x]^2\,\text{Log}\left[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2\,d-e}\right)\,e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\right]-$$

$$\pi\,\text{ArcSinh}[c\,x]\,\text{Log}\left[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2\,d-e}\right)\,e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\right]+$$

$$4\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{ArcSinh}[c\,x]\,\text{Log}\left[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2\,d-e}\right)\,e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\right]+$$

$$i\,\text{ArcSinh}[c\,x]^2\,\text{Log}\left[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2\,d-e}\right)\,e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\right]+$$

$$\pi\,\text{ArcSinh}[c\,x]\,\text{Log}\left[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2\,d-e}\right)\,e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\right]-$$

$$4\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{ArcSinh}[c\,x]\,\text{Log}\left[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2\,d-e}\right)\,e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\right]-$$

$$i\,\text{ArcSinh}[c\,x]^2\,\text{Log}\left[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2\,d-e}\right)\,e^{\text{ArcSinh}[c\,x]}}{\sqrt{e}}\right]+$$

$$i\,\text{ArcSinh}[c\,x]^2\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcSinh}[c\,x]}}{i\,c\sqrt{d}-\sqrt{-c^2\,d+e}}\right]-$$

$$i\,\text{ArcSinh}[c\,x]^2\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcSinh}[c\,x]}}{-i\,c\sqrt{d}+\sqrt{-c^2\,d+e}}\right]-i\,\text{ArcSinh}[c\,x]^2$$

$$\text{Log}\left[1-\frac{\sqrt{e}\,e^{\text{ArcSinh}[c\,x]}}{i\,c\sqrt{d}+\sqrt{-c^2\,d+e}}\right]+i\,\text{ArcSinh}[c\,x]^2\,\text{Log}\left[1+\frac{\sqrt{e}\,e^{\text{ArcSinh}[c\,x]}}{i\,c\sqrt{d}+\sqrt{-c^2\,d+e}}\right]-$$

$$\pi\,\text{ArcSinh}[c\,x]\,\text{Log}\left[1+\frac{i\left(c\sqrt{d}-\sqrt{c^2\,d-e}\right)\left(c\,x+\sqrt{1+c^2\,x^2}\right)}{\sqrt{e}}\right]-$$

$$4\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{ArcSinh}[c\,x]\,\text{Log}\left[1+\frac{i\left(c\sqrt{d}-\sqrt{c^2\,d-e}\right)\left(c\,x+\sqrt{1+c^2\,x^2}\right)}{\sqrt{e}}\right]+$$

$$i\,\text{ArcSinh}[c\,x]^2\,\text{Log}\left[1+\frac{i\left(c\sqrt{d}-\sqrt{c^2\,d-e}\right)\left(c\,x+\sqrt{1+c^2\,x^2}\right)}{\sqrt{e}}\right]+$$

$$\pi\,\text{ArcSinh}[c\,x]\,\text{Log}\left[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2\,d-e}\right)\left(c\,x+\sqrt{1+c^2\,x^2}\right)}{\sqrt{e}}\right]+$$

$$4\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{ArcSinh}[c\,x]\,\text{Log}\left[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2\,d-e}\right)\left(c\,x+\sqrt{1+c^2\,x^2}\right)}{\sqrt{e}}\right]-$$

$$i\,\text{ArcSinh}[c\,x]^2\,\text{Log}\left[1+\frac{i\left(-c\sqrt{d}+\sqrt{c^2\,d-e}\right)\left(c\,x+\sqrt{1+c^2\,x^2}\right)}{\sqrt{e}}\right]+$$

$$\pi\,\text{ArcSinh}[c\,x]\,\text{Log}\left[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2\,d-e}\right)\left(c\,x+\sqrt{1+c^2\,x^2}\right)}{\sqrt{e}}\right]-$$

$$4\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{ArcSinh}[c\,x]\,\text{Log}\left[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2\,d-e}\right)\left(c\,x+\sqrt{1+c^2\,x^2}\right)}{\sqrt{e}}\right]-$$

$$i\,\text{ArcSinh}[c\,x]^2\,\text{Log}\left[1-\frac{i\left(c\sqrt{d}+\sqrt{c^2\,d-e}\right)\left(c\,x+\sqrt{1+c^2\,x^2}\right)}{\sqrt{e}}\right]-$$

$$\pi\,\text{ArcSinh}[c\,x]\,\text{Log}\left[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2\,d-e}\right)\left(c\,x+\sqrt{1+c^2\,x^2}\right)}{\sqrt{e}}\right]+$$

$$4\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{c\sqrt{d}}{\sqrt{e}}}}{\sqrt{2}}\right]\,\text{ArcSinh}[c\,x]\,\text{Log}\left[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2\,d-e}\right)\left(c\,x+\sqrt{1+c^2\,x^2}\right)}{\sqrt{e}}\right]+$$

$$i\,\text{ArcSinh}[c\,x]^2\,\text{Log}\left[1+\frac{i\left(c\sqrt{d}+\sqrt{c^2\,d-e}\right)\left(c\,x+\sqrt{1+c^2\,x^2}\right)}{\sqrt{e}}\right]-$$

$$2\,i\,\text{ArcSinh}[c\,x]\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcSinh}[c\,x]}}{i\,c\sqrt{d}-\sqrt{-c^2\,d+e}}\right]+$$

$$2\,i\,\text{ArcSinh}[c\,x]\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcSinh}[c\,x]}}{-i\,c\sqrt{d}+\sqrt{-c^2\,d+e}}\right]+$$

$$2\,i\,\text{ArcSinh}[c\,x]\,\text{PolyLog}\left[2,-\frac{\sqrt{e}\,e^{\text{ArcSinh}[c\,x]}}{i\,c\sqrt{d}+\sqrt{-c^2\,d+e}}\right]-$$

$$2\,i\,\text{ArcSinh}[c\,x]\,\text{PolyLog}\left[2,\frac{\sqrt{e}\,e^{\text{ArcSinh}[c\,x]}}{i\,c\sqrt{d}+\sqrt{-c^2\,d+e}}\right]+$$

$$2\,i\,\text{PolyLog}\left[3,\frac{\sqrt{e}\,e^{\text{ArcSinh}[c\,x]}}{i\,c\sqrt{d}-\sqrt{-c^2\,d+e}}\right]-2\,i\,\text{PolyLog}\left[3,\frac{\sqrt{e}\,e^{\text{ArcSinh}[c\,x]}}{-i\,c\sqrt{d}+\sqrt{-c^2\,d+e}}\right]-$$

$$\left.\left.2\,i\,\text{PolyLog}\left[3,-\frac{\sqrt{e}\,e^{\text{ArcSinh}[c\,x]}}{i\,c\sqrt{d}+\sqrt{-c^2\,d+e}}\right]+2\,i\,\text{PolyLog}\left[3,\frac{\sqrt{e}\,e^{\text{ArcSinh}[c\,x]}}{i\,c\sqrt{d}+\sqrt{-c^2\,d+e}}\right]\right)\right)$$

Problem 44: Result unnecessarily involves higher level functions and more than

twice size of optimal antiderivative.

$$\int \frac{a+b\,\text{ArcSinh}[c\,x]}{\left(d+e\,x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 70 leaves, 6 steps):

$$\frac{x\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{d\,\sqrt{d+e\,x^2}}-\frac{b\,\text{ArcTanh}\left[\frac{\sqrt{e}\,\sqrt{1+c^2\,x^2}}{c\,\sqrt{d+e\,x^2}}\right]}{d\,\sqrt{e}}$$

Result (type 6, 166 leaves):

$$\frac{1}{\sqrt{d+e\,x^2}}x\left(\left(2\,b\,c\,x\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2\,x^2,-\frac{e\,x^2}{d}\right]\right)\Big/\left(\sqrt{1+c^2\,x^2}\left(-4\,d\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2\,x^2,-\frac{e\,x^2}{d}\right]+x^2\left(e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2\,x^2,-\frac{e\,x^2}{d}\right]+c^2\,d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2\,x^2,-\frac{e\,x^2}{d}\right]\right)\right)\right)+\frac{a+b\,\text{ArcSinh}[c\,x]}{d}\right)$$

Problem 45: Result unnecessarily involves higher level functions.

$$\int \frac{a+b\,\text{ArcSinh}[c\,x]}{\left(d+e\,x^2\right)^{5/2}}\,dx$$

Optimal (type 3, 146 leaves, 7 steps):

$$-\frac{b\,c\,\sqrt{1+c^2\,x^2}}{3\,d\,\left(c^2\,d-e\right)\,\sqrt{d+e\,x^2}}+\frac{x\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,d\,\left(d+e\,x^2\right)^{3/2}}+\frac{2\,x\,\left(a+b\,\text{ArcSinh}[c\,x]\right)}{3\,d^2\,\sqrt{d+e\,x^2}}-\frac{2\,b\,\text{ArcTanh}\left[\frac{\sqrt{e}\,\sqrt{1+c^2\,x^2}}{c\,\sqrt{d+e\,x^2}}\right]}{3\,d^2\,\sqrt{e}}$$

Result (type 6, 235 leaves):

$$\frac{1}{3\,d^2\,\left(d+e\,x^2\right)^{3/2}}\left(-\frac{b\,c\,d\,\sqrt{1+c^2\,x^2}\,\left(d+e\,x^2\right)}{c^2\,d-e}+a\,x\,\left(3\,d+2\,e\,x^2\right)+\left(4\,b\,c\,d\,x^2\,\left(d+e\,x^2\right)\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2\,x^2,-\frac{e\,x^2}{d}\right]\right)\Big/\left(\sqrt{1+c^2\,x^2}\left(-4\,d\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2\,x^2,-\frac{e\,x^2}{d}\right]+x^2\left(e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2\,x^2,-\frac{e\,x^2}{d}\right]+c^2\,d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2\,x^2,-\frac{e\,x^2}{d}\right]\right)\right)\right)+b\,x\,\left(3\,d+2\,e\,x^2\right)\,\text{ArcSinh}[c\,x]\right)$$

Problem 46: Result unnecessarily involves higher level functions.

$$\int \frac{a + b \,\text{ArcSinh}[c\, x]}{\left(d + e\, x^2\right)^{7/2}} \, dx$$

Optimal (type 3, 227 leaves, 8 steps):

$$-\frac{b\, c\, \sqrt{1+c^2 x^2}}{15\, d\, \left(c^2 d-e\right) \left(d+e\, x^2\right)^{3/2}}-\frac{2\, b\, c\, \left(3\, c^2 d-2\, e\right) \sqrt{1+c^2 x^2}}{15\, d^2 \left(c^2 d-e\right)^2 \sqrt{d+e\, x^2}}+\frac{x\, \left(a+b\, \text{ArcSinh}[c\, x]\right)}{5\, d\, \left(d+e\, x^2\right)^{5/2}}+$$

$$\frac{4\, x\, \left(a+b\, \text{ArcSinh}[c\, x]\right)}{15\, d^2 \left(d+e\, x^2\right)^{3/2}}+\frac{8\, x\, \left(a+b\, \text{ArcSinh}[c\, x]\right)}{15\, d^3 \sqrt{d+e\, x^2}}-\frac{8\, b\, \text{ArcTanh}\left[\frac{\sqrt{e}\, \sqrt{1+c^2 x^2}}{c\, \sqrt{d+e\, x^2}}\right]}{15\, d^3 \sqrt{e}}$$

Result (type 6, 308 leaves):

$$\left(-\frac{b\, c\, d^2 \sqrt{1+c^2 x^2}\, \left(d+e\, x^2\right)}{c^2 d-e}-\frac{2\, b\, c\, d\, \left(3\, c^2 d-2\, e\right) \sqrt{1+c^2 x^2}\, \left(d+e\, x^2\right)^2}{\left(-c^2 d+e\right)^2}+a\, x\, \left(15\, d^2+20\, d\, e\, x^2+8\, e^2 x^4\right)+\right.$$

$$\left(16\, b\, c\, d\, x^2 \left(d+e\, x^2\right)^2 \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -c^2 x^2, -\frac{e\, x^2}{d}\right]\right) \Big/ \left(\sqrt{1+c^2 x^2}\right.$$

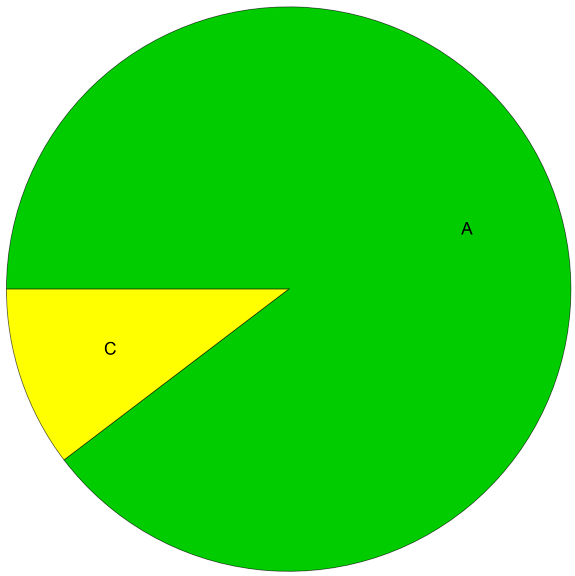
$$\left(-4\, d\, \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -c^2 x^2, -\frac{e\, x^2}{d}\right]+x^2 \left(e\, \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, -c^2 x^2, -\frac{e\, x^2}{d}\right]+\right.\right.$$

$$\left.\left.\left.c^2 d\, \text{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, -c^2 x^2, -\frac{e\, x^2}{d}\right]\right)\right)\right)+$$

$$\left.b\, x\, \left(15\, d^2+20\, d\, e\, x^2+8\, e^2 x^4\right) \text{ArcSinh}[c\, x]\right) \Big/ \left(15\, d^3 \left(d+e\, x^2\right)^{5/2}\right)$$

Summary of Integration Test Results

58 integration problems



A - 52 optimal antiderivatives

B - 0 more than twice size of optimal antiderivatives

C - 6 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts